A Simheuristic Algorithm for the Uncapacitated and Stochastic Hub Location Problem

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Abstract

Telecommunication network-design problems have gained attention over the last decades. Among them, the problem looking for a network design that minimizes total costs, while satisfying users' demands is well-known. This is the particular case of the Hub Location Problem (HLP). In data communications, a hub is a place of convergence where data arrive from one or more directions and are forwarded in one or more other directions. In this work-in-progress we proposed the application of a simheuristic algorithm for solving the stochastic uncapacitated HLP in which uncertainty is associated to transportation costs.

1 Introduction

Telecommunication networks present appealing application areas for diverse outlooks of location optimization. A popular optimization problem with practical applications to telecommunication networks is the Facility Location Problem (FLP), which involves deciding the position of an undetermined number of facilities to minimize the sum of: *(i)* the setup cost of these facilities; and *(ii)* the cost related to serving the customers from them. The basic version of this problem is the Uncapacitated Facility Location Problem (UFLP), which assumes that the capacity of each facility is virtually unlimited or, at least, far beyond the expected demand. The facility location models for telecommunications networks are concerned with the location of concentrators and the assignment of terminals to these concentrators. In general, location problems in telecommunications are NP-hard combinatorial problems.

One of the most paradigmatic location problems in telecommunications is the Hub Location Problem (HLP) (see Figure 1). Besides telecommunications and computing network, these problems are receiving increased attention in transportation and logistics. There are several kinds of HLPs: the number of hubs may or may not be established beforehand, the node allocation may be either single or multiple, there may be constraints on the capacities, etc. Analogous to the UFLP, there exists the Uncapacitated HLP (UHLP) in which the number of hubs is not specified but there are fixed costs for establishing hubs on the nodes and there are no capacities.

Most of the models applied in HLP literature are considering deterministic parameters, while stochastic and uncertain cases are more realistic. In fact, some attributes of HLP such as demand and setup costs for the hubs have inherent uncertainity in real-world applications. However, to the best of our knowledge, there are few published articles related to stochastic HLPs [1, 2, 4, 6, 7, 8]. Recently, de Armas *et al.* [3], introduced a novel simheuristic algorithm [5] designed to deal with the stochastic version of the UFLP (SUFLP), where the service costs are considered stochastic. With this in mind, this work proposes a modified version of the former method in order to solve the UHLP with stochastic transportation costs (UHLP-STC) i.e. the *transportation cost is modeled as a random variable*. The remainder of this abstract is organized as follows: Section 2 explains the proposed implementation for solving the UHLP-STC based on the simheuristic developed by de Armas *et al.*. Section 3 analyzes some numerical results. Finally, conclusions are drawn in Section 4.

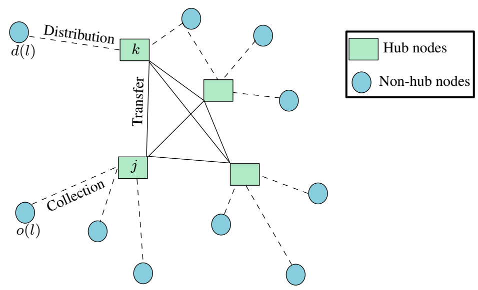

Figure 1: Illustrative example of the Hub Location Problem.

2 Solving Methodology

We consider the UHLP-STC in which transportation costs are stochastic. The mathematical model was provided by [2]. Given that hub nodes are fully interconnected, every path between an origin and a destination node will contain at least one hub. For this reason, paths between two nodes are of the form $(o(l), j, k, d(l))$, where (j, k) is the ordered pair of hubs to which $o(l)$ and $d(l)$ are allocated, respectively. Therefore, the unit transportation cost of routing commodity l along path $(o(l), j, k, d(l))$ is given by $F_{jkl} = \mathcal{X} c_{o(l)j} + \tau c_{jk} + \delta c_{kd(l)}$ which it is assumed as stochastic, where $\mathcal{X}$, τ and δ represent the collection, transfer and distribution costs along the path as depicted in Figure 1.

Due to the successful results presented in [3] for solving the UFLP, this work aims to follow the same methodology used there. As in the former proposal, a heuristic for the deterministic version of the UHLP is necessary as initial stage for developing the simheuristic algorithm for the UHLP-STC. Then, the previous heuristic is integrated into an ILS metaheuristic framework. This allows for improving the quality of the generated solution whenever more computing time is allowed. Finally, the simheuristic algorithm [5] is obtained by integrating the metaheuristic with Monte Carlo simulation techniques.

The logic behind this development process is: first, optimal solutions for the deterministic UHLP are obtained by implementing a fast savings-based heuristic able to provide reasonably good solutions in milliseconds. Then, this heuristic is extended to a competitive ILS-based approach, which is able to provide near-optimal solutions in short time. Finally, a simheuristic algorithm for the stochastic version of the problem, based on the integration of Monte Carlo simulation is built. Algorithm 1 presents the pseudo-code of the proposed methodology.

Algorithm 1: UHLP-STC

```
Generate a initSol by implementing savingList
Generate a baseSol by applying localSearch(initSol)
bestSol ← baseSol
fastSimulation(bestSol)
while (nIter ≤ maxIter) do
    Generate a newSol by perturbating baseSol
    newSol ← localSearch(newSol)
    fastSimulation(newSol)
    if (newSol better than baseSol) then          // newSol improves baseSol
        baseSol ← newSol
        if (newSol better than bestSol) then      // newSol improves bestSol
            bestSol ← newSol
            insert(poolBestSol, bestSol)
        end
    end
end
for (sol ∈ poolBestSol) do
    deepSimulation(sol)
end
return poolBestSol
```

3 Preliminary Computational Experiments

In order to present initial empirical results to assess the behavior of the simheuristic algorithm, we first focus on an implementation regarding a simplified version of the deterministic problem, the UHLP with multiple assignments (UHLPMA) in which the costs between an origin to the first hub, transfer between the hubs and final distribution from the last hub to the destination location are assumed to be null. With this in mind, the largest instances in the literature have been used to represent a challenging scenario. We have generated a set of benchmark instances using the procedure described in [3]. For validation purposes, Table 1 shows the mean of the total costs obtained over a set of $100,000$ runs for each of the following solutions: *(i)* optimal solution *det* for the deterministic version when applied to the stochastic environment, *(ii)* the solution *min-avg* with minimum expected cost found by the simheuristic algorithm and *(iii)* the solution *min-q3* which refers to the minimum quartile to the observed costs after multiple executions of a solution. In a general perspective, the solution *min-avg* is quite similar to solution *det*, which contribute to validate the proposed methodology. However, the solution *min-q3* seems fairly different when compared to the other solutions.

Instance	Solution		
	det	*min-avg*	*min-q3*
500_10	3082981,19	2926702,40	2933333,76
1000_10	5601942,04	5460931,66	5547130,71
1500_10	7697299,35	7854292,20	8239254,07
2000_10	9823853,71	9989287,84	9388958,98
2500_10	12080157,15	12042008,99	11709263,65
3000_10	13942310,57	13696784,88	13184613,68

Table 1: Comparison of the solution obtained by the simheuristic approach.

4 Conclusions and Future Work

The complexity of telecommunication networks as well as the enormous increasing traffic flow present challenging location-based problems. The introduction of uncertainty leads to a much greater compliance with the real-life problems encountered not only in telecommunications but in the areas of logistics and transports. In this abstract, a simheuristic algorithm for solving the uncapacitated hub location problem with stochastic transportation cost (UHLP-STC) has been proposed. For the case of the deterministic problem UHLPMA under some mild conditions, the results contribute to validate the methodology proposed. To extend this work-in-progress, an extension of this algorithm will be considered in order to solve the UHPL-STC considering the unit cost of routing commodity l along the path $(o(l), j, k, d(l))$ which includes the collection, transfer and distribution costs.

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